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Influence of local wind speed and direction on wind power dynamics - Application to offshore very short-term forecasting

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Abstract

Wind power time series usually show complex dynamics mainly due to non-linearities related to the wind physics and the power transformation process in wind farms. This article provides an approach to the incorporation of observed local variables (wind speed and direction) to model some of these effects by means of statistical models. To this end, a benchmarking between two different families of varying-coefficient models (regime-switching and conditional parametric models) is carried out. The case of the offshore wind farm of Horns Rev in Denmark has been considered. The analysis is focused on one-step ahead forecasting and a time series resolution of 10 minutes. It has been found that the local wind direction contributes to model some features of the prevailing winds, such as the impact of the wind direction on the wind variability, whereas the non-linearities related to the power transfor-

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mation process can be introduced by considering the local wind speed. In both cases, conditional parametric models showed a better performance than the one achieved by the regime-switching strategy. The results attained reinforce the idea that each explanatory variable allows the modelling of different underlying effects in the dynamics of wind power time series.

Keywords: Energy systems modelling, Forecasting, Wind power, Offshore, Varying-coefficient

1. Introduction

The explosive growth of installed wind power over the last 10 years combined with the progressive liberalization of electrical markets have given rise to some new challenges related to wind energy [1]. Special attention has turned towards wind power forecasting, concerning the activity of two agents: wind power producers need to provide accurate information about their energy production in order to take part in the electrical market and the Transmission System Operators (TSO's) need to keep the stability of the electrical system also facing fluctuations on the generation side. In fact, when a certain penetration of wind generation is attained, uncertainties about the evolution of the wind may force the TSO to switch-off a certain number of wind farms, even when the resource is available. These facts represent a clear limitation for wind power penetration, specially considering the ambitious development plans of the offshore industry for the next years [2]. However, accurate forecasts for horizons varying from few minutes to several days could help to mitigate the impact of the inherent uncertainty of the wind. As a result, the last decade has witnessed a rapid growth in the field of short-term wind

18 power forecasting, for both statistical and physical approaches [3, 4, 5, 6, 7].

19 In this article we focus on the very-short term case, typically being based
20 on a prediction horizon of some minutes to few hours. For such prediction
21 horizons, it is generally accepted that statistical time series based models are
22 more accurate than physical models, the latter ones being more appropriate
23 for horizons beyond several hours [3, 5, 8]. The objective of statistical time
24 series based models is to learn and replicate the dynamics shown by the tem-
25 poral evolution of certain variables (such as the power output time series)
26 under the hypothesis that these dynamics reflect different underlying effects
27 of the wind power conversion process. Some of these effects would be at-
28 mospheric processes occurring at different scales [9], the electrical conversion
29 carried out by the wind turbine, the wake effect generated by nearby wind
30 turbines, etc. [10, 11].

31 The present work aims to disentangle some of the effects mentioned above
32 by means of a set of available local measurements and an appropriate sta-
33 tistical model. Linear statistical models are characterized by their simplicity
34 and reliability. Even though both wind speed and wind power time series
35 show highly non-linear dynamics, several methodologies have been proposed
36 based on a linear approach (see [12, 13, 14, 15, 16, 17, 18] among others).
37 On the other hand, non-linear approaches are usually based on non paramet-
38 ric models such as Artificial Neural Networks [19], which does not permit a
39 clear interpretation of the underlying processes being modelled. We focus
40 on a non-linear approach based on varying-coefficient models [20] by gen-
41 eralising linear Autoregressive models (AR). The basic structure of an AR
42 model considers the forecasted value as a linear combination of past values

by employing fixed weights (see Eq. 6). The main idea is to replace these constant parameters by functions that take into account local observations such as wind speed and direction. This allows the modelling of dependencies in the time series dynamics based on other explanatory variables in a simple way.

Regime-switching autoregressive models are a particular case of varying-coefficient models that consider AR coefficients as constant piece-wise functions. In this case, the considered time series is supposed to evolve shifting between clearly differentiated dynamics (called regimes). These kind of models give rise to a new problem because regimes have to be identified and delimited in some sense [21]. If the shift between regimes is modelled as a function of lagged values of a time series, the process is called observable. This is the case of Threshold Autoregressive Open Loop (TARSO) models [22, 23, 24]. A different approach is considered by Markov Switching Autoregressive models (MSAR), where the current regime is a non-observable process following a first order Markov chain [25, 26, 27, 28, 29, 30].

On the other hand, Conditional Parametric Autoregressive models (CPARX) consider the AR coefficients as smooth functions of some explanatory variables [31, 32, 33]. There exist several approaches to estimate these coefficient-functions (see [34] and references therein). For example, the locally weighted linear regression introduced by Cleveland and Devlin [35] was applied in the design of the Danish Wind Power Prediction Tool WPPT4 [36]. In that case, the AR coefficients were modelled as a function of the forecasted wind speed and direction provided by physical Numerical Weather Prediction (NWP) model.

68 To the authors' knowledge, there is relatively little research concerning
69 regime-switching models and conditional parametric models that take into
70 account on-line available data such as local wind speed and direction. Thus,
71 in this article we propose a benchmark between the two mentioned families
72 of models (regime-switching and conditional parametric models) in order to
73 clarify how this information can be added so as to model specific features
74 of the wind power time series dynamics. Three reference models are also
75 considered: Persistence, linear AR and MSAR models. Table 1 summarizes
76 different regime-switching and conditional parametric models reviewed in the
77 literature, as well as those considered in this study.

78 The paper is structured as follows: In Section 2 a theoretical descrip-
79 tion of the models considered in this article is presented. In Section 3 the
80 database of the case study is described, the offshore wind farm of Horns Rev.
81 The application of the models are detailed in Section 4, organized in four
82 subsections: (i) Description of the reference models, (ii) Modelisation of the
83 local wind direction influence, (iii) Modelisation of the local wind speed influ-
84 ence and (iv) Combining the effects of both local wind speed and direction.
85 Results are presented and discussed in Section 5. Finally, the main findings
86 of the article are summarized in Section 6.

87 **2. Theoretical description of the models**

88 From now, $\{y_t\}$, $t = 1, \dots, N$ represents a discrete time series with N
89 observations of averaged wind power production. $\{x_t\}$, $x_t \in \mathbb{R}$, $t = 1, \dots, N$
90 is a discrete time series with N observations of a certain exogenous variable.
91 Additionally, \mathcal{Y}_T and \mathcal{X}_T denote vectors gathering the first T values of the

92 corresponding time series, e.g. $\mathcal{Y}_T = (y_1, \dots, y_T)$. $\{y_t\}$ is supposed to follow
 93 a stochastic process like:

$$y_t = f(\mathcal{Y}_{t-k}, \mathcal{X}_{t-k}, \Theta) + \varepsilon_t \quad (1)$$

94 f provides the deterministic component of y_t as a function of a certain
 95 set of parameters Θ and the available observations \mathcal{Y}_{t-k} and \mathcal{X}_{t-k} , k being
 96 the prediction horizon. $\{\varepsilon_t\}$ is a white noise process, that represents the
 97 noise of the stochastic process. The purpose of each model considered is
 98 to determine a certain function \hat{f} , this function being a proposal for the
 99 unknown deterministic component of the process. Nevertheless, there are
 100 some considerations that establish a common framework for the development
 101 of every model considered here. First, only the case of one-step ahead is
 102 considered, thus, $k = 1$. Moreover, the white noise is assumed to follow a
 103 centred Gaussian distribution with standard deviation σ , i.e., $\varepsilon_t \sim \mathcal{N}(0, \sigma^2)$.
 104 Hence, a certain model forecasts the value y_t , denoted with \hat{y} , as follows:

$$\hat{y}_t = E(y_t | \mathcal{Y}_{t-1}, \mathcal{X}_{t-1}, \Theta) = \hat{f}(\mathcal{Y}_{t-1}, \mathcal{X}_{t-1}, \Theta) \quad (2)$$

105 where $E(a|b)$ represents the expectation of the statistical variable a given b .

106 In order to estimate the set of parameters of a statistical model, Θ , the
 107 minimisation problem given by Eq. (3) has to be considered along with a
 108 score function. In this work we use the quadratic error function of Eq. (4)
 109 evaluated over a set of historical data (training-set) with N_{train} samples.

$$\hat{\Theta} = \underset{\Theta}{\operatorname{argmin}} \mathcal{E}(\Theta) \quad (3)$$

$$\mathcal{E}(\Theta) = \sum_{t=p+1}^{N_{train}} (y_t - \hat{y}_t)^2 \quad (4)$$

110 In the following subsections, the linear reference models are described first
 111 (Persistence and linear AR), then a non-linear reference model (the MSAR
 112 model, a regime-switching model without exogenous variables) and finally,
 113 TARSO and CPARX models, which comprise a set of varying-coefficient
 114 models that take into account the local wind direction and the local wind
 115 speed as explanatory variables.

116 *2.1. Linear reference models: Persistence and autoregressive*

117 Persistence is the most common reference forecasting method for predic-
 118 tion horizons up to 4-6 hours, due to the characteristic time of changes in the
 119 atmosphere [37]. A clear advantage of this model is that neither a parameter
 120 estimation nor exogenous variables are needed. Persistence states that the
 121 forecasted value at time t is the last available value:

$$\hat{y}_t = y_{t-1} \quad (5)$$

122 An AR(p) is an order- p linear model that considers \hat{y}_t as a weighted sum
 123 of the previous p observed values:

$$\hat{y}_t = \theta_0 + \sum_{i=1}^p \theta_i \cdot y_{t-i} \quad (6)$$

124 In this case, given a certain order p , the set of parameters Θ gathers the
 125 $p + 1$ AR coefficients. This set will be noted as $\Theta_{AR(p)}$

$$\Theta_{AR(p)} = \{\theta_0, \theta_1, \dots, \theta_p\} \quad (7)$$

126 Since varying-coefficient models proposed in this article are obtained by
 127 generalising a linear AR model, comparison between them reveals the im-
 128 provement obtained just related to the consideration of changing regimes or
 129 smooth dependencies.

130 *2.2. Non-linear reference model: Markov-Switching Autoregressive Models*

131 The first generalisation of linear AR models considered are the MSAR
 132 models. These models assume that a time series evolves switching between
 133 different autoregressive dynamics (called regimes). The shift between regimes
 134 is considered as a non observable process, which means that it cannot be de-
 135 termined by lagged values of the time series. Pinson et al. [29] demonstrated
 136 that MSAR models provided better results than other regime-switching mod-
 137 els for two case studies of off-shore wind power forecasting, mainly because
 138 these models manage to capture more complex dynamics in regime-switching
 139 than when considering the regime as an observable process. Hence, MSAR
 140 models represent a suitable option to evaluate the improvement related to
 141 regime-switching hypothesis in the absence of exogenous variables. For this
 142 reason, MSAR models are here considered as the third reference model.

143 Let us consider that a time series evolves according to a certain num-
 144 ber, r , of different regimes. The current regime at time t is given by the
 145 discrete state variable s_t , $t = 1, \dots, N$, $s \in \{1, \dots, r\}$. The shift between
 146 regimes is governed by a first order Markov chain, hence the probability
 147 $p(s_t | \mathcal{S}_{t-1}, \mathcal{Y}_{t-1}) = p(s_t | s_{t-1})$. These probabilities are collected in the so-
 148 called transition matrix P , where $P_{ij} = p(s_t = j | s_{t-1} = i)$. Since the process
 149 is considered unobservable, $\{s_t\}$ is hidden and has to be inferred from avail-
 150 able data through the Hamilton filter introduced in Hamilton [38]. Each

151 regime j , $j = 1, \dots, r$, is supposed to follow an $AR(p)$ process with coefficients
 152 $\Theta_{AR(p)}^{(j)} = \{\theta_0^{(j)}, \dots, \theta_p^{(j)}\}$ and standard deviation $\sigma^{(j)}$. The set of parameters of
 153 the MSAR model, Θ_{MSAR} , gathers the transition matrix, the AR coefficients
 154 and the standard deviation for each regime:

$$\Theta_{MSAR} = \{P, \Theta_{AR(p)}^{(1)}, \dots, \Theta_{AR(p)}^{(r)}, \sigma^{(1)}, \dots, \sigma^{(r)}\} \quad (8)$$

155 As an example, Figure 1 illustrates the filtered probabilities of the current
 156 regime along with the power output time series for a short window time. It
 157 can be seen how the filtered probabilities balance depending on the level of
 158 fluctuations. During periods with missing-data, the transition matrix deter-
 159 mines a smooth exponential convergence to the so-called ergodic probabilities
 160 (the probabilities of being in a certain regime at an arbitrary date).

161 MSAR models can be formulated in two different ways [39]: the Intercept-
 162 Form (MSAR-IF, Eq. 9) and the Mean Adjusted Form (MSAR-MAF, Eq.
 163 10).

$$y_{t,IF}^{(s_t)} = \theta_0^{(s_t)} + \sum_{i=1}^p \theta_i^{(s_t)} \cdot y_{t-i} + \varepsilon_t^{(s_t)} \quad (9)$$

$$y_{t,MAF}^{(s_t)} - \mu_0^{(s_t)} = \sum_{i=1}^p \phi_i^{(s_t)} \cdot (y_{t-i} - \mu_0^{(s_{t-i})}) + \varepsilon_t^{(s_t)} \quad (10)$$

164 When no regimes are considered, both forms are equivalent by considering
 165 $\phi_i = \theta_i, \forall i > 0$ and $\mu_0 = \theta_0 / (1 - \sum_{i=1}^p \theta_i)$. Nevertheless, MSAR-IF and
 166 MSAR-MAF model different underlying dynamics [39].

167 2.3. TARSO models

168 Open Loop Threshold Autoregressive models are a kind of regime-switching
 169 model where the current regime s_t is assessed by a predefined function of the
 170 available observations of exogenous variables, $s_t = s_t(\mathcal{X}_{t-1})$. Hence the pro-
 171 cess is called observable. Usually, only a certain lag of x_t is considered,
 172 $s_t = g(x_{t-lag})$. In that case, regimes are settled by a certain number of
 173 thresholds, $l_0, l_1, l_2, \dots, l_r$, that divide the space spanned by $\{x_t\}$ in r subsets,
 174 called S_j , $j = 1, \dots, r$ from now. Then, $x_{t-lag} \in S_j \Leftrightarrow l_{j-1} \leq x_{t-lag} < l_j$.

175 In this article only the previous lag of the exogenous variable is considered
 176 in assessing regimes. An AR process is assumed in each regime. For the sake
 177 of simplicity, all the AR processes will have the same order p . The model is
 178 given by:

$$y_t = \theta_0^{(s_t)} + \sum_{i=1}^p \theta_i^{(s_t)} \cdot y_{t-i} + \varepsilon_t^{(s_t)} \quad (11)$$

$$s_t = \begin{cases} 1, & x_{t-1} \in S_1 \\ 2, & x_{t-1} \in S_2 \\ \dots & \\ r, & x_{t-1} \in S_r \end{cases}$$

179 With the mentioned hypothesis, the implementation of a TARSO model
 180 gives rise to three questions: (i) what is the number, r , of regimes considered,
 181 (ii) what is the optimal value for the set of thresholds $\mathbf{l} = \{l_0, \dots, l_r\}$ and (iii)
 182 what AR order p to choose.

183 Modelling a wind power time series with the described TARSO model

184 implies that the wind farm output has clearly differentiated dynamics de-
 185 pending on the value of some observed variable. For example, in the case
 186 of the wind direction (wd), a different behaviour of the wind power time
 187 series would be expected depending on the local wind direction observed at
 188 the moment of making the forecasting, wd_{t-1} . If wd_{t-1} crosses one of the
 189 thresholds given by \mathbf{l} , then there is an abrupt change on the AR process that
 190 provides the forecast \hat{y}_t .

191 2.4. CPARX Models

192 Conditional parametric models are characterized by a smooth dependence
 193 of their coefficients with a certain variable. In particular, the CPARX models
 194 generalize an AR model by letting the coefficients depend on available obser-
 195 vations of exogenous variables, $\theta_i = \theta_i(\mathcal{X}_{t-1})$. As in the preceding case, only
 196 the previous lag of the exogenous variable will be considered. The model is
 197 given by:

$$y_t = \theta_0(x_{t-1}) + \sum_{i=1}^p \theta_i(x_{t-1}) \cdot y_{t-i} + \varepsilon_t \quad (12)$$

198 A central point is how to define the coefficient-functions $\theta_i(x_{t-1})$. They
 199 can be estimated with non-parametric techniques from historical data or by
 200 means of a parametric function [40, 41]. In this work, the latter case will be
 201 considered.

202 Modelling a wind power time series with a CPARX model implies that
 203 the wind farm output dynamic is expected to change smoothly depending
 204 on the value of some observed variable x_{t-1} . For example, in the case of the
 205 wind speed, (ws), the observed local value ws_{t-1} fixes at each time step the

206 AR process (through the coefficient-functions $\theta_i(ws_{t-1})$) that provides the
207 forecast \hat{y}_t .

208 3. Description of the data

209 The data considered originates from the offshore wind farm located at
210 Horns Rev, off the west coast of Denmark. This wind farm has a rated
211 power of 160 MW. Measurements of wind power output, wind speed and
212 direction are available for each wind turbine, with a one-second sample rate.
213 10-minute resolution time series are derived by averaging raw data. At least
214 75% of the data within an interval has to be considered as valid in order
215 to consider the averaged value also valid. The averaging process assures
216 that the fast fluctuations related to the turbulent nature of the wind have
217 been filtered. The period considered ranges from 16th February 2005 to 31st
218 January 2006, consisting of 50,400 data points with 8,790 missing data. The
219 data-base has been divided into the following 3 sets:

- 220 · Training-set, from 16th February to 31st May 2005: the parameters
221 of the models are estimated considering this data set by solving the
222 minimisation problem given by Eq. (3).
- 223 · Validation-set, from 1st June to 31st August 2005: the forecasts pro-
224 vided by the trained models are evaluated during this second period.
225 By doing this, it is possible to assess the generalization capabilities
226 of each model, which means that a certain model trained over a first
227 period keeps its prediction performances over a different time period.

228 · Test-set, from 1st September 2005 to 31st January 2006: a benchmark
229 analysis between validated models is carried out based on their fore-
230 casting performance in this period.

231 It should be notice that the division of the data-set does not permit
232 models to capture seasonalities during the training process, which covers
233 almost four months. This seasonalities are expected to be present in wind
234 power time series considering the seasonal variability of wind at Horns Rev
235 observed in Vincent et al. [42]. However, it does not necessarily imply that
236 the optimal models would dramatically change from one month to another.
237 In any case, the optimisation of the models taking into account seasonal
238 variations would require several years of data (not available for this work) and
239 the implementation of models with time-varying parameters being adaptively
240 estimated. In this regard, the implementation of adaptive MSAR models was
241 addressed in [30].

242 4. Application of the models

243 In this section, the implementation of the models considered in Section
244 2 in the case of data described in Section 3 is presented. The section is
245 divided in four subsection on different alternatives about the explanatory
246 variables considered. Each model is trained with different structures (con-
247 cerning for example the AR order and the definition of regimes). The optimal
248 parametrisation of each model was chosen regarding the generalisation capa-
249 bilities across the validation-set. The performance of the models is evaluated
250 in terms of the Normalized Root Mean Square Error (*NRMSE*) and the
251 percentage of Improvement Over Persistence (*IoP*), defined as follows:

$$NRMSE = \frac{1}{P_N} \cdot \sqrt{\sum_{t=p+1}^N \frac{(y_t - \hat{y}_t)^2}{N - p}} \quad (13)$$

$$IoP(\%) = 100 \cdot \frac{NRMSE_0 - NRMSE}{NRMSE_0} \quad (14)$$

where P_N is the rated power of the wind farm and $NRMSE_0$ is the $NRMSE$ obtained with Persistence . Both criteria are suggested in Madsen et al. [37], which includes a broad overview of ways to evaluate wind power prediction methods.

4.1. Reference models

This subsection deals with the implementation of the reference models described in subsections 2.1 (Persistence and linear AR) and 2.2 (MSAR models). As previously mentioned, Persistence does not have free parameters to be estimated. Thus, the performance of this model is evaluated in a straightforward way. This is not the case for the linear AR models, since the appropriate AR order p and the set of parameters $\Theta_{AR(p)}$ need to be estimated. For a given value of p , $\Theta_{AR(p)}$ is estimated by means of the Yule-Walker equations (available in several works, e.g. [43]) over the training period. Then, the evaluation of the trained models over the validation-set allowed the optimal value of $p = 3$ to be identified.

Next, both MSAR-IF and MSAR-MAF architectures are employed to model the wind power time series of Horns Rev. In order to estimate Θ_{MSAR} , the Expectation-Maximization algorithm introduced in Dempster et al. [44] and further described in Hamilton [45] is applied (for further details, see [38,

271 46]). In the case of the MSAR-IF form, three regimes were identified with
 272 the following set of parameters:

Regime	θ_0	θ_1	θ_2	θ_3	σ
$s_t = 1$	0.01	1.24	-0.47	0.19	0.0573
$s_t = 2$	0.04	1.21	-0.24	0.00	0.0004
$s_t = 3$	0.00	1.45	-0.50	0.04	0.0075

$$P = \begin{bmatrix} 0.77 & 0.02 & 0.21 \\ 0.11 & 0.73 & 0.16 \\ 0.27 & 0.03 & 0.70 \end{bmatrix}$$

273 On the other hand, the MSAR-MAF model identified the two following
 274 regimes:

Regime	μ_0	ϕ_1	ϕ_2	ϕ_3	σ
$s_t = 1$	0.52	1.25	-0.46	0.18	0.0565
$s_t = 2$	0.53	1.38	-0.45	0.08	0.0121

$$P = \begin{bmatrix} 0.91 & 0.09 \\ 0.07 & 0.93 \end{bmatrix}$$

275 In both cases, the regimes were identified by sorting different levels of
 276 fluctuations, i.e., different values for $\sigma^{(i)}$, the standard deviation of the noise.

277 4.2. Modelling the influence of the local wind direction

278 In this subsection, the inclusion of the local wind direction into both
 279 TARSO and CPARX models is detailed. In order to get some clues about

the dependence of wind power on wind direction, a preliminary analysis has been carried out. This would eventually suggest restrictions to the design of appropriate varying-coefficient models, e.g. the number of regimes and the shape of the parameter functions. Then, both the TARSO(wd) model and the CPARX(wd) model are implemented.

4.2.1. Preliminary analysis

The central idea is to train a linear AR model over a subset of the training data. The subset is given by the membership of the previous wind direction lag to a certain sector over the wind rose. The set of AR coefficients, Θ_{AR} , and the $NRMSE$ obtained characterize the dynamic of the wind power output related to this particular sector. Then, by sliding smoothly the orientation of the sector and repeating the process, one observes the impact of wind direction on wind power dynamics.

Let us consider a main direction α_0 and a sector width h . The AR(p) model for this sector is given by:

$$\begin{cases} \hat{y}_t = \theta_0 + \sum_{i=1}^p \theta_i \cdot y_{t-i} \\ \forall t : wd_{t-1} \in \alpha_0 \pm h/2 \end{cases}$$

The estimation of this model provides specific values for $\Theta_{AR(p)}$ and $NRMSE$, related to α_0 . Figure 2 illustrates the dependence of α_0 on $\theta_{AR(p)}$ and the $NRMSE$, when considering the case for $p = 2$ and $h = 90^\circ$. The following conclusions were derived from the previous analysis, where the considered values for p ranged from 1 to 5: (i) AR coefficients showed a certain dependence on α_0 for any value of p . This dependence is smooth sinus-shaped. (ii) The highest $NRMSE$ (thus, the lowest predictability) is

302 related to 270°-310° directions. (iii) The relationship between the *NRMSE*
 303 and α_0 shows a similar tendency in both the training-set and the validation-
 304 set. Hence, the influence of the wind direction learnt from historical data
 305 seems to be representative enough to model future behaviour.

306 4.2.2. *TARSO models based on a wind direction criterion: TARSO(wd)*

307 The previous analysis highlights different predictability levels, depending
 308 on the wind direction. Furthermore, there seems to be a high predictability
 309 orientation (E-SE), a low one (W-NW) and intermediate transitions. This
 310 fact suggests a low number of regimes to be considered a priori.

311 The TARSO model was introduced in Eq. (11). In this particular case,
 312 regime thresholds \mathbf{l} will be related to wind direction sectors as follows: let
 313 us consider a main direction α_0 and a certain width sector h . For the sake
 314 of simplicity, the same h will be considered for every sector. The wind rose
 315 can be split in $r = 360^\circ/h$ sectors (the considered widths in the preliminary
 316 analysis assures that the number of sectors is a natural number between 2
 317 and 8) by defining the following thresholds:

$$\begin{aligned} l_j &= \alpha_0 + \frac{2j-1}{2} \cdot h, \quad j = 1, \dots, r \\ l_0 &= l_r \end{aligned}$$

318 This procedure provides the definition of \mathbf{l} and r , given values of α_0 and
 319 h . Once the sectors have been defined, AR coefficients can be estimated for
 320 each regime once more by means of the Yule-Walker equations. Figure 3
 321 shows the *NRMSE* obtained in the validation-set as a function of p and r ,
 322 when considering the optimal orientation α_0 obtained. It can be noted that

the model with the best generalization capability was obtained for the case of $p = 3$. In the same way, it does not seem to be worth increasing the number of regimes further than 3. In relation to the orientation sectors, Figure 4 illustrates the best ones for the six AR(3) models. It can be seen that the sectors are placed in such a way that the above mentioned low predictability orientation ($W-NW$) tends to form an independent regime, independently of the number of regimes considered.

The TARSO(wd) model that showed the best performance in the validation-set was:

$$\hat{y}_t = \begin{cases} 0.00 + 1.36 \cdot y_{t-1} - 0.51 \cdot y_{t-2} + 0.14 \cdot y_{t-3}, & s_t = 1 \\ 0.01 + 1.40 \cdot y_{t-1} - 0.54 \cdot y_{t-2} + 0.13 \cdot y_{t-3}, & s_t = 2 \\ 0.00 + 1.19 \cdot y_{t-1} - 0.43 \cdot y_{t-2} + 0.23 \cdot y_{t-3}, & s_t = 3 \end{cases}$$

The regimes were given by:

$$s_t = \begin{cases} 1, & wd_{t-1} \in [-41^\circ, 79^\circ) \\ 2, & wd_{t-1} \in [79^\circ, 199^\circ) \\ 3, & wd_{t-1} \in [199^\circ, 319^\circ) \end{cases}$$

4.2.3. CPARX models based on a wind direction criterion: CPARX(wd)

The description of CPARX models in Subsection 2.4 highlights that the crucial point is how to define the coefficients as a function of a certain exogenous variable. Considering the previous preliminary analysis, a sinus-shaped dependence is proposed:

$$\hat{y}_t = \theta_0(wd_{t-1}) + \sum_{i=1}^p \theta_i(wd_{t-1}) \cdot y_{t-i} \quad (15)$$

$$\theta_i(wd_{t-1}) = a_i + b_i \cdot \cos(wd_{t-1} - \phi_0), \quad i = 0, \dots, p \quad (16)$$

338 a_i being the mean level of the i 'th AR coefficient and b_i being the amplitude
 339 of the dependence of θ_i on the wind direction. Then, for a given value of p ,
 340 the set of parameters is formed by:

$$\Theta_{CPARX} = \{a_0, \dots, a_p, b_0, \dots, b_p, \phi_0\} \quad (17)$$

341 Θ_{CPARX} is estimated in accordance with Eq. (3). As in the previous case,
 342 the best performance in the validation-set was achieved for the case of $p = 3$.
 343 Figure 5 collects the AR coefficients for the AR model, the TARSO(wd)
 344 model and the CPARX(wd) model.

345 4.3. Modelling the influence of the local wind speed

346 Following a similar methodology, this subsection focuses on how the local
 347 wind speed can be used to define regimes or smooth dependences in the wind
 348 power time series dynamics. A preliminary analysis between the predicted
 349 variable and the wind speed is firstly performed. Then, the TARSO(ws)
 350 model and the CPARX(ws) model are obtained.

351 4.3.1. Preliminary analysis

352 Let us consider the interval of wind speeds $I = [ws_0 - h/2, ws_0 + h/2]$.
 353 An AR(p) model is trained taking into account only those data that satisfy
 354 at time t the condition $ws_{t-1} \in I$. For a certain h , the AR coefficients and
 355 the $NRMSE$ obtained are related to the wind speed ws_0 . Then, the interval

356 I slides over the spanned space of the wind speed in order to reveal how
 357 the time series dynamic and the predictability vary with ws_0 . The following
 358 conclusions were obtained, where the considered values for p ranged from 1
 359 to 5: (i) The AR coefficients show a certain dependence on the wind speed.
 360 This dependence is close to be linear in a substantial part of the wind speed
 361 range, as is shown in the Figure 6 (case $p = 2$, $h = 4$ m/s). (ii) The $NRMSE$
 362 tends to be higher for high wind speeds, showing a maximum at a wind speed
 363 of around $10 - 12$ m/s. However, a decrease in the $NRMSE$ is observed for
 364 wind speeds beyond the nominal wind speed (at which the output power
 365 is constant up to the cut-off wind speed). (iii) A similar tendency of the
 366 relationship between $NRMSE$ and wind speed has been found for both the
 367 training-set and the validation-set (see Figure 6). This fact suggests that
 368 the data sets are representative enough to consider this information valid for
 369 future time periods.

370 4.3.2. *TARSO models based on a wind speed criterion: $TARSO(ws)$*

371 The prior analysis reveals that a regime-switching model can be imple-
 372 mented in order to catch different predictability levels, though a low regimes
 373 number is suggested from Figure 6. In this case, the optimisation process
 374 considers the threshold values, \mathbf{l} , as parameters to be estimated. Then, for a
 375 certain number of regimes, r , and the AR order p , the set of parameters to
 376 estimate is given by:

$$\Theta_{TARSO} = \{\Theta_{AR^{(1)}}, \dots, \Theta_{AR^{(r)}}, \mathbf{l}\} \quad (18)$$

377 Θ_{TARSO} is estimated by means of a numerical algorithm based on the
 378 criterion given by Eq. (3). Two and three regimes have been proposed

379 with AR orders going from 1 to 5. In all the cases, the $AR(3)$ showed
 380 the best performance in the validation-set (see Figure 7). Furthermore, the
 381 two-regimes model was slightly better than the three-regimes one. Figure 8
 382 illustrates the power curve depicted under the optimised regimes. In both
 383 cases, the thresholds obtained seems to be related to the shape of the power
 384 curve. First, considering two regimes lead to a threshold of around 10 m/s
 385 near the inflexion point. This value splits up the power curve in two regions:
 386 (i) the first one is characterized by a convex relationship between the wind
 387 speed and the output power. In an ideal case, this relationship is a cubic
 388 polynomial given by $P = \frac{1}{2}\rho C_p A v^3$, where ρ is the density of air, C_p is the
 389 power coefficient, A is the area swept by the rotor blades and v is the wind
 390 speed. (ii) The second part is characterized by a concave relationship, since
 391 the output power has to be limited by the rated power of the wind turbine.
 392 On the other hand, considering three regimes leads to a division clearly based
 393 on the slope of the power curve: two regimes for the two flat regions (for low
 394 and high wind speeds) and a third one for the steep part.

395 The TARSO(ws) model with best generalisation capabilities was:

$$\hat{y}_t = \begin{cases} 0.00 + 1.33 \cdot y_{t-1} - 0.50 \cdot y_{t-2} + 0.18 \cdot y_{t-3}, & s_t = 1 \\ -0.02 + 1.22 \cdot y_{t-1} - 0.39 \cdot y_{t-2} + 0.18 \cdot y_{t-3}, & s_t = 2 \end{cases}$$

396 The regimes were given by:

$$s_t = \begin{cases} 1, & ws_{t-1} < 10.08 \\ 2, & ws_{t-1} \geq 10.08 \end{cases}$$

397 4.3.3. CPARX models based on a wind speed criterion: CPARX(ws)

398 In this case, a linear dependence between AR coefficients and the last
 399 available data of wind speed ws_{t-1} is proposed (Eqs. (19) and (20)). This is
 400 partially supported by the preliminary analysis: even though this hypothesis
 401 does not seem to be accurate for low and high wind speeds, Figure 6 reveals
 402 that it is the case for a substantial part of the wind speed range.

$$\hat{y}_t = \theta_0(ws_{t-1}) + \sum_{i=1}^p \theta_i(ws_{t-1}) \cdot y_{t-i} \quad (19)$$

$$\theta_i(ws_{t-1}) = a_i + b_i \cdot (ws_{t-1}), \quad i = 0, \dots, p \quad (20)$$

403 a_i being the i 'th AR coefficient at null wind speed and b_i being the slope of
 404 the dependence of θ_i on the wind speed. The set of parameters is now given
 405 by $\Theta_{CPARX} = \{a_0, \dots, a_p, b_0, \dots, b_p\}$ and estimated in accordance with Eq.
 406 (3). The minimisation process has been carried out for several AR orders,
 407 $p = 1, 2, \dots, 5$, giving $p = 3$ the optimal value in terms of generalisation
 408 capabilities. Figure 9 collects the AR coefficients obtained as a function of the
 409 wind speed for the AR model, the TARSO(ws) model and the CPARX(ws)
 410 model.

411 4.4. Combining both effects: CPARX(wd, ws)

412 Results concerning the incorporation of local wind direction and local
 413 wind speed in varying-coefficient models will be discussed in Section 5. How-
 414 ever, at this point, it is worth noting that CPARX models showed a better
 415 performance than TARSO models when modelling the effect of the considered
 416 explanatory variable (see Figure 11). Additionally, each exogenous variable

417 seems to provide information about different effects. In base of this hypoth-
 418 esis, the following CPARX model considering both wind speed and wind
 419 direction is proposed:

$$\hat{y}_t = \theta_0(wd_{t-1}, ws_{t-1}) + \sum_{i=1}^p \theta_i(wd_{t-1}, wd_{t-1}) \cdot y_{t-i} \quad (21)$$

420

$$\begin{aligned} \theta_i(wd_{t-1}, ws_{t-1}) &= a_i + b_i \cdot \cos(wd_{t-1} - \phi_0) \\ &+ c_i \cdot (ws_{t-1}), \quad i = 0, \dots, p \end{aligned} \quad (22)$$

421 The set of parameters to be estimated is $\Theta_{CPARX} = \{a_0, \dots, a_p, b_0, \dots, b_p, c_0, \dots, c_p, \phi_0\}$.
 422 In this case, the best model obtained was for an AR order of $p = 4$. The
 423 coefficient-functions $\theta_i(wd_{t-1}, ws_{t-1})$ are now surfaces that replicates the
 424 same trends found in the previous sections. As an example, the case of
 425 θ_1 is illustrated in Figure 10.

426 5. Results

427 This section gathers the results obtained over the test-set, when the op-
 428 timal parametrisation of each model obtained in Section 4 is considered.

429 Globally, the improvements over Persistence ranged from almost 4% to
 430 more than 5.5% (see Fig. 11). This represents a good performance, since Per-
 431 sistence is traditionally difficult to improve on for a prediction horizon of 10
 432 minutes. With regard to the reference models and in accordance with the pre-
 433 vious studies [29, 30], improvements in very-short term point-forecasting can
 434 be attained when considering several regimes under the absence of other ex-
 435 planatory variables. In particular, MSAR models were able to capture shifts
 436 between non-observed meteorological states, delivering information about

437 wind power fluctuations and providing a better performance than Persistence
438 and linear AR models.

439 The models taking into account exogenous variables overcome the refer-
440 ence models. Regarding the influence of the local wind direction, a similar
441 relationship between this variable and the AR parameters was identified by
442 the TARSO(wd) and the CPARX(wd) models, as shown in Figure 5. In par-
443 ticular, given that Persistence can be considered as a particular case of AR
444 model with $\theta_1 = 1$ and $\theta_i = 0, \forall i > 1$, both TARSO(wd) and CPARX(wd)
445 models were likely to become globally closer to Persistence for wind direc-
446 tions related to the W-NW sector, characterized by a low predictability (the
447 only exception being θ_3 , which experiences a small increment for the men-
448 tioned wind directions). Additionally, a smooth dependence of the wind
449 power dynamics on the local wind direction was found to be preferable to
450 considering different regimes (though special attention was paid to track the
451 optimal number of sectors and their orientation) given the IoP of 4.98% and
452 4.66% respectively. Similar conclusions were obtained when the local wind
453 speed was considered as an exogenous variable: both models TARSO(ws)
454 and CPARX(ws) became globally closer to Persistence (with the only ex-
455 ception of θ_3 , which remains almost constant) for high wind speeds (Figure
456 9) characterized by a lower predictability, and a smooth dependence of the
457 coefficient-functions on the wind speed provided a better result than the
458 regime-switching strategy (an IoP of 4.82% compared to 4.58%).

459 In general, the models that took into account the wind direction at-
460 tained slightly better results than those including the wind speed. This
461 was also found when considering the results depicted monthly (Tables 2 and

3), the only exception being the month of January. However, in both a globally and a monthly basis, the best performance was clearly attained by the CPARX(wd,ws). This model attained a global IoP of 5.72%, which represents almost the addition of the single improvements obtained by the CPARX(wd) and the CPARX(ws) models with respect to the AR model. This finding is particularly significant as it supports the notion that each explanatory variable gives information about effects of a different nature.

5.1. Further discussion

It was found that the incorporation of the wind direction as an explanatory variable leads to an appreciable improvement of the prediction performance. It could be due to the fact that the proposed models managed to capture some influence of the local wind direction on the wind power time series dynamics. Vincent et al. [42] related the influence of the wind direction on the wind variability at Horns Rev to synoptic scale forcings combined with the location of the wind farm with respect to the shore. In particular, a high wind variability was observed for Westerly winds. According to Akhmatov [47], the implementation of the models of Subsection 4.2 evidences that these effects are propagated to the wind power time series. As mentioned above, it is interesting to note that a smooth dependence of the wind power dynamics on the local wind direction was preferable to a regime switching strategy. This could be explained by taking the following considerations: the present study is focused on an offshore wind farm, characterized by a flat topography with a uniform-clustered distribution of the wind turbines over a squared area. Hence, for this wind farm configuration no obstacle is introducing directional aerodynamic disturbances and, additionally, wind turbine

wakes are likely to have a weaker impact on the dependence between the wind power and the local wind direction compared to the case of a single row wind farm configuration. Even though some works [10, 11] suggest a considerable influence of the wakes for very narrow sectors around the wind turbines line direction, this seems to be too specific to be relevant from a statistical point of view (at least with the models considered in this work). Our results suggest that the influence of the local wind direction on the wind power dynamics was likely to be related to synoptic conditions rather than microscale effects. However, microscale effect could become predominant in other study cases. Modelling the influence of the local wind direction in wind farms located in complex terrain, where topographic obstacles and non-homogeneity of the terrain introduce strong directional dependences on the power production, could require other AR coefficient-functions, instead of the sinus-shaped ones proposed here. Furthermore, wind farms with a non-squared distribution of wind turbines, for instance row-configured wind farms, could even require a regime switching strategy, since the wind turbine wakes would affect dramatically the performance of the wind farm for certain wind directions. In any case, further research on complex terrain and different configuration of wind farms would be required for confirmation.

On the other hand, when the local wind speed was considered as an exogenous variable, the optimisation of the models were likely to be related to the characteristics of the non-linear power transformation process. Considering that the power curve represents a non-linear transformation from wind speed to wind power, the slope of this curve provokes an amplification/reduction effect of the wind speed fluctuations. It has a direct impact on the out-

512 put power dynamics, causing a dependence between the wind speed and the
 513 predictability of the output wind power. Hence, the improvement obtained
 514 could be due to the fact that the wind speed was employed as a signal about
 515 this non-linear effect. The regime-switching strategy provided thresholds of
 516 wind speed that divide the power curve into particular parts (convex-concave
 517 for the case of 2 regimes and low-high-low amplification level for the case of
 518 3 regimes, see Figure 8). For the case of the conditional parametric model,
 519 a linear relationship between the AR coefficients and the wind speed seemed
 520 to be appropriate for a greater part of the wind speed range. However, the
 521 saturation effect of the output power related to extreme wind speeds (close
 522 to zero or above the nominal wind speed) has not been addressed. Future
 523 work could deal with this topic by considering the Generalized Logit trans-
 524 formation described in Pinson [48] or the so-called ‘break-point models’, a
 525 special subclass of varying-coefficient models that combine both CPARX and
 526 TARSO models (see the closing discussion in Hastie and Tibshirani [49]).

527 **6. Conclusions**

528 We have presented a study focused on modelling the influence of local
 529 wind speed and direction on the dynamics of a wind power time series. With
 530 this purpose, a benchmark between several varying-coefficient models for 10
 531 minute-ahead forecasting was carried out. The models are built by general-
 532 ising the conventional linear AR structure, following two approaches: regime
 533 switching models and conditional parametric models. By comparing the ac-
 534 curacy of the models, findings about the most suitable statistical approach
 535 were also obtained.

536 It was found that local measurements of both wind speed and direction
537 provide useful information for a better comprehension of the wind power time
538 series dynamics, at least when considering the case of the very-short term
539 forecasting. In particular, the results suggest that different effects can be
540 modelled depending on the considered explanatory variable: the local wind
541 direction contributes to model some features of the prevailing winds, such as
542 the impact of the wind direction on the wind variability, whereas the non-
543 linearities related to the power transformation process can be introduced
544 by considering the local wind speed. Additionally, for our particular case
545 study, it was found that the conditional parametric models outperforms a
546 regime-switching strategy.

547 It is interesting to note that the influence of both local wind speed and
548 direction were modelled under the assumption of observable processes, and
549 that only the last observation was taken into account. This study highlights
550 two main lines for further research: the first one is to consider non-observable
551 processes based on local observations, by incorporating exogenous variables
552 whether in the transition matrix or in the definition of the AR coefficients
553 of MSAR models. The second one is to include previous lags of the local
554 observations in order to get a model sensitive to the evolution of the con-
555 sidered exogenous variable. By doing this, it would be possible to explore
556 new effects that condition the dynamics of the output wind power time series
557 (e.g. abrupt changes in local wind direction related to certain meteorological
558 conditions).

559 Finally, the models here presented could be upgraded by letting the co-
560 efficients vary smoothly with time so as to capture seasonal variabilities of

561 wind power dynamics due to climatological effects and the decrease of the
562 wind turbine performance.

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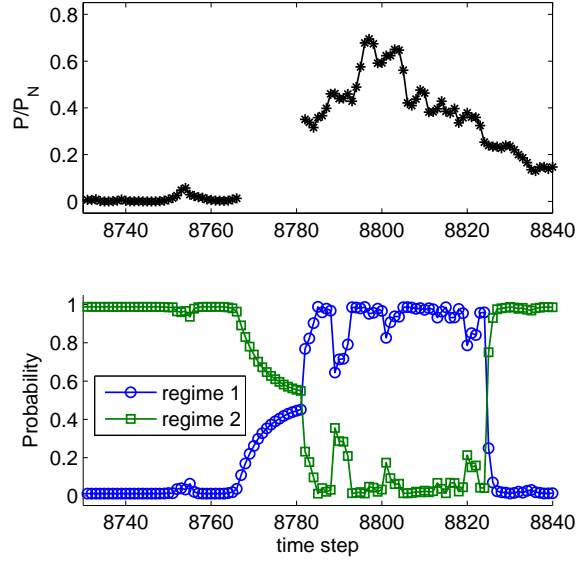


Figure 1: Filtered probabilities of the current regime provided by the MSAR model during periods with missing data. P/P_N represents the output power (P) normalized with the rated power of the wind farm (P_N).

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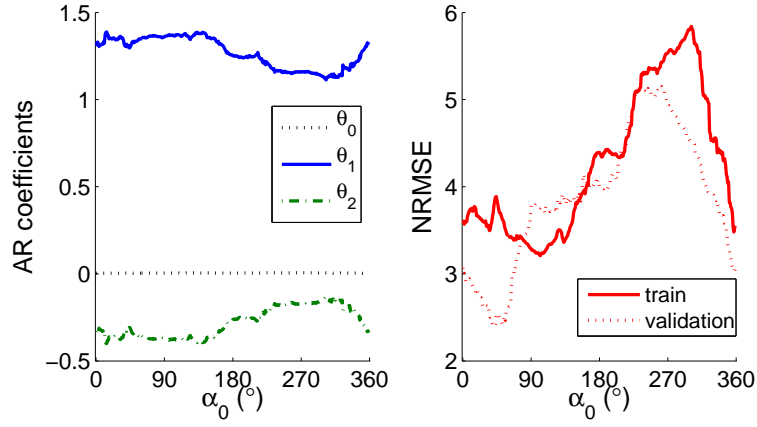


Figure 2: Dependence of the AR coefficients (left) and $NRMSE$ in $\%P_N$ (right) with local wind direction. Case for AR order $p = 2$

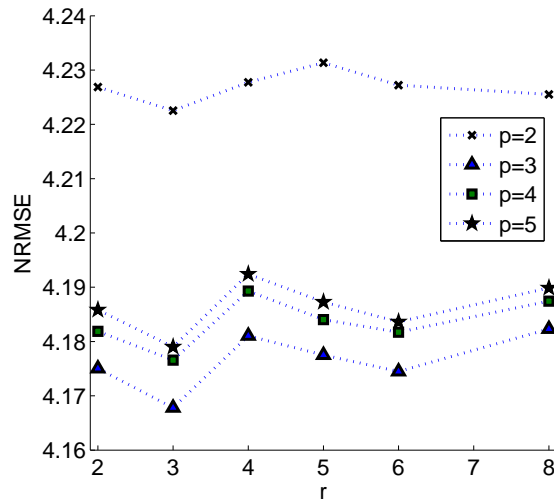


Figure 3: $NRMSE$ (in $\%P_N$) of TARSO(wd) over the validation-set, as a function of the number of regimes, r , and the AR order, p . Results for $p = 1$, layout of the picture due to a higher $NRMSE$

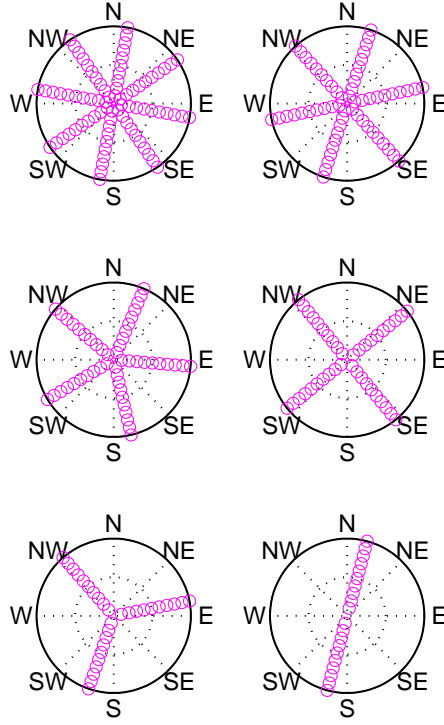


Figure 4: Optimal orientation of the sectors depending on the number of regimes considered in a $\text{TARSO}(wd)$ model, case for AR order $p = 3$

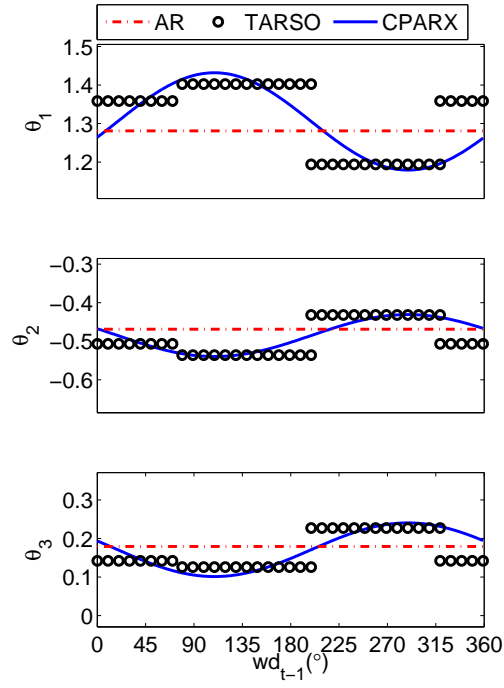


Figure 5: Dependence of the AR coefficients with local wind direction for AR, TARSO(wd) and CPARX(wd) models (Θ_0 is omitted, since it is very close to zero for every model)

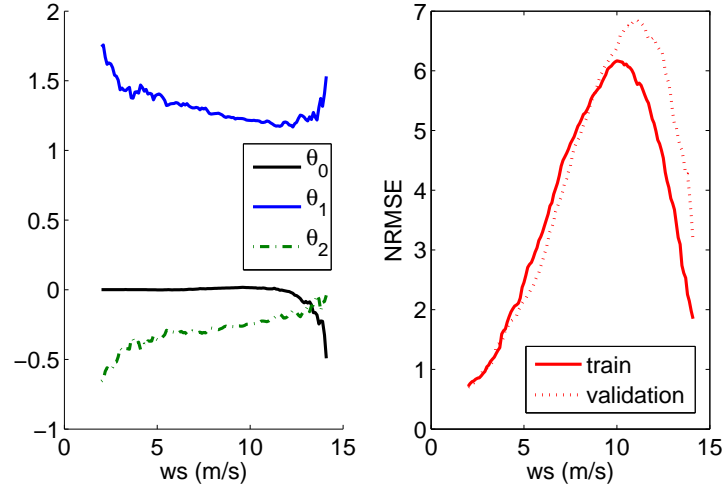


Figure 6: Dependence of the AR coefficients and $NRMSE$ (in $\%P_N$) with local wind speed. Case for AR order $p = 2$

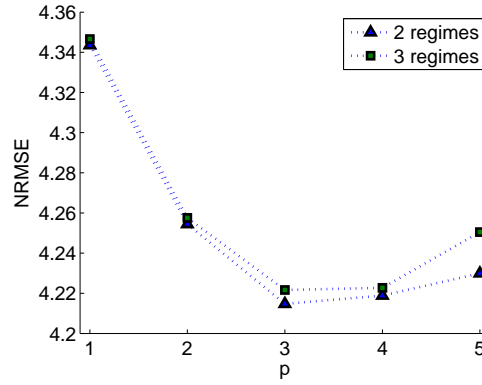


Figure 7: $NRMSE$ (in $\%P_N$) of $TARSO(ws)$ over the validation-set, as a function of the number of regimes, r , and the AR order, p

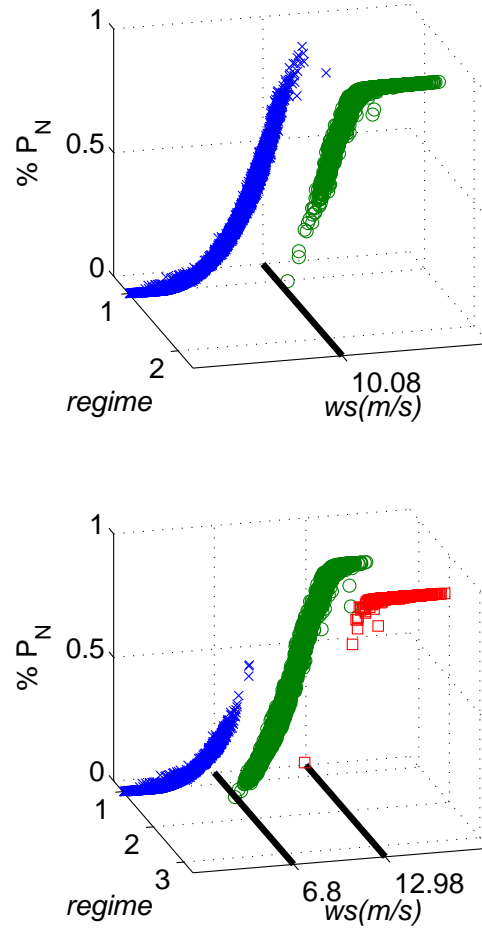


Figure 8: Optimal splitting of the power curve for TARSO(ws) models

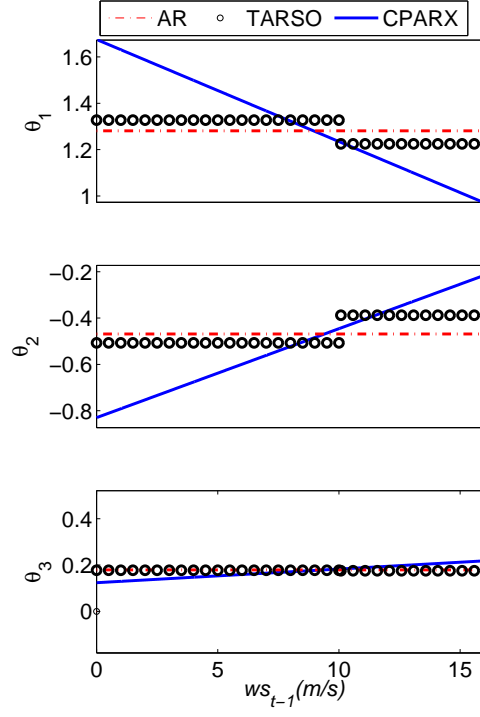


Figure 9: Dependence of the AR coefficients for AR, TARSO(ws) and CPARX(ws) models. (θ_0 is omitted, since it is very close to zero for every model)

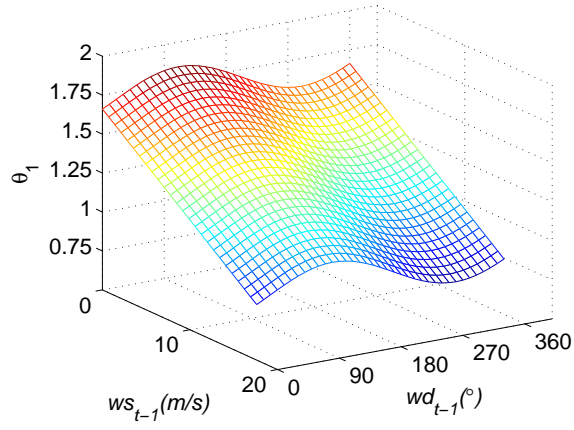


Figure 10: θ_1 as a function of local wind direction and local wind speed for the $\text{CPARX}(wd,ws)$ model

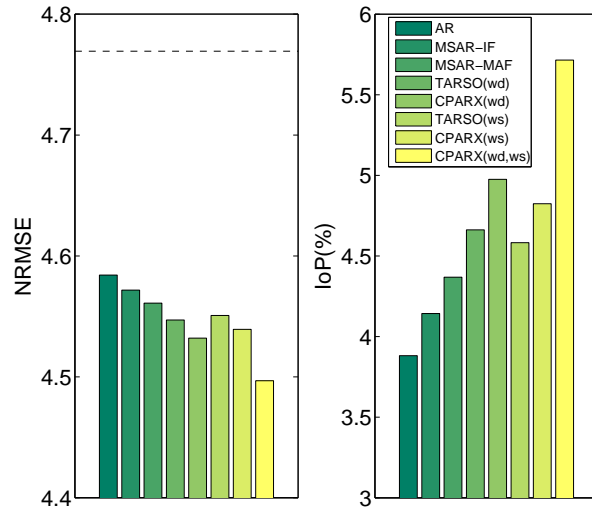


Figure 11: $NRMSE$ (in $\%P_N$) and IoP for the test-set. Dashed line of the figure on the left refers to the $NRMSE$ of Persistence

Constant coefficients	
Persistence ^{1,2,3} , AR ^{2,3,4} , ARMA ^{1,5,6}	
Varying coefficients	
R-S (Obs)	STAR ¹ , SETAR ¹ , TARSO ⁷
R-S (Non-Obs)	MSAR ^{1,2,8}
C-P	CPARX ^{3,9}

Table 1: Summary of models applied in some studies related to short-term wind and wind power forecasting. In bold, models considered in the present study. **R-S**: Regime-Switching, **C-P**: conditional parametric, **Obs**: Observable process. ¹Pinson et al. [29], ²Pinson and Madsen [30], ³Pinson [48], ⁴Brown et al. [12], ⁵De Giorgi et al. [16], ⁶Erdem and Shi [17] ⁷Tastu et al. [23], ⁸Ailliot [27], ⁹Nielsen et al. [36]

	September	October	November	December	January
Persistence	4.66	4.16	6.25	4.76	4.07
AR	4.44	3.96	6.03	4.42	3.98
MSAR-IF	4.43	3.95	5.97	4.47	3.97
MSAR-MAF	4.41	3.96	5.95	4.41	4.00
TARSO(<i>wd</i>)	4.42	3.92	5.94	4.37	3.99
CPARX(<i>wd</i>)	4.41	3.92	5.91	4.37	3.97
TARSO(<i>ws</i>)	4.44	3.93	5.94	4.39	3.97
CPARX(<i>ws</i>)	4.42	3.94	5.92	4.37	3.94
CPARX(<i>wd,ws</i>)	4.41	3.91	5.82	4.35	3.93

Table 2: *NRMSE* depicted monthly. The two lowest values in each column are given in bold fonts. The overall results are gathered in Figure 11

	September	October	November	December	January
AR	4.73	4.89	3.54	7.19	2.33
MSAR-IF	4.74	5.04	4.43	6.10	2.50
MSAR-MAF	5.30	4.81	4.80	7.46	1.80
TARSO(<i>wd</i>)	5.07	5.72	4.87	8.17	2.07
CPARX(<i>wd</i>)	5.34	5.71	5.41	8.34	2.54
TARSO(<i>ws</i>)	4.72	5.65	4.89	7.89	2.57
CPARX(<i>ws</i>)	5.07	5.39	5.16	8.14	3.30
CPARX(<i>wd,ws</i>)	5.37	5.96	6.78	8.68	3.54

Table 3: *IoP* depicted monthly. The two highest values in each column are given in bold fonts. The overall results are gathered in Figure 11